Analysis of hydrologic characteristics of a basin for snowmelting period by filter separation AR method

By

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Abstract In this paper, the characteristics of hydrologic basins as well as snowfall are derived only from daily snowmelt runoffs data. This is a sort of inverse hydrologic problem.

First, time series of daily snowmelt runoffs are separated into componets of the longer period and the shorter period by numerical filters whose cutoff frequencies is determined from the coherence gap in snowfall- snowmelt runoff because the daily data can be considered to be a stationary stochastic process.

By assuming the daily snowfall to be of a white noise, the time series of each component are fitted to the AR model from which the hydrologic response characteristics (unit hydrograph for each subsystem) are determined.

Time series of the daily snowfall are inversely generated from the original time series of the daily snowmelt runoff and the unit hydrograph of each component determined above.

Finally, the estimated snowfall is compared with observed effective snowfall which is obtained by multiplying the observed snowfall by runoff ratio (total observed snowfall / total snowmelt runoff).

1. INTRODUCTION

The increase of a heap of reliable hydrologic data as well as the advancements in the science and technology of statiscal and stochastic data analysis , supplimented with electronic equipments and machines, have stimulated hydrologists and hydraulicians to the flourishing proposals of hydrologic data analysis. Here, the filter separation AR method (Hino and Hasebe ,1979 and 1981), which the authors' proposed, attempt to derive the hydrologic characteristics of a basin for snowmelting period and the inverse estimation of effective snowfall time series from the daily snowmelt runoff data alone; i.e. the hydrologic inverse proplem. The present method utilizes the stochastic approach. However, it is reinforced by the understanding of the physical and thus deterministic process of hydrologic systems.

2. RECAPITULATION OF FILTER SEPARATION AR METHOD

The followings are the recapitulation of the filter separation AR method of hydrologic analysis from daily snowmelt runoff data alone. The method is developed based on the snowfall property that snowfalls observed in *unit of day* may be treated as a white noise whose auto-correlation is a Dirac's delta function. Accordingly, a snowmelt runoff component such as the longer period component or the shorter period one may be seen as the output from an AR(auto-regressive) model driven by a white noise.

2.1 Separation of snowmelt runoff into components by numerical filter

Time series of daily snowmelt runoff is divided two snowmelt runoff components through the high frequency cutoff numerical filters. The cutoff frequency of the numerical filter is determined both from the AR model fitting and coherence and phase between snowfall (input) and snowmelt runoff (output) (Hino,1977).

The coherence and phase is given by equ.(1).

$$Coh^{2}(f) = P_{xy}(f)^2/(P_x(f)*P_y(f))$$
$$Phase(f) = \tan^{-1}(Q_{xy}(f)/K_{xy}(f)) \quad (1)$$

where Px(f) is power spectrum of input, Py(f) is power spectrum of output, Pxy(f) is cross spectrum, Kxy(f) is cospectrum and Qxy(f) is quadrature spectrum.

The cutoff frequency (Hino and Hasebe,1984) to separate snowmelt runoff time series is given by equ.(2).

$$fc = 1/Tc \quad (2)$$

where fc is cutoff frequency and Tc is the time characteristic of separation.

The high frequency cutoff filtering of daily snowmelt runoff y(t) is performed by equ.(3).

$$y^{(1)}(t) = a\Sigma\omega_k \cdot y(t-k) \quad (3)$$

where $y^{(1)}(t)$ is the filtered longer period runoff ($\triangle t = 1$) and a is a weighting factor.

$$\omega_k = \frac{C_0\exp[-C_1(k\triangle t)/2] \cdot \sinh[(C_1^2-C_0)^{1/2}(k\triangle t)]}{(C_1^2-C_0)^{1/2}}$$
$$= 0$$
$$for\ k=0,1,2,\ldots,N\ and\ k=-1,-2,\ldots \quad (4)$$

The values of C_0 and C_1 in equ.(4) are given by

$$C_0 = (\delta/Tc)^2$$
$$C_1 = \delta^2/Tc$$

where δ is a damping factor.

2.2 Hydrologic characteristics of subsystem

Time series of each separated runoff component is fitted to the AR model given by equ.(5).

$$y_t^{(l)} = a_1 y_{t-1}^{(l)} + a_2 y_{t-2}^{(l)} + \cdots\cdots + a_p y_{t-p}^{(l)} + \varepsilon_t^{(l)} \quad (5)$$

where ε is a white noise and $l = 1$ and 2.

From the AR coefficients thus determind $a^{(l)}$, the unit impulse response function $h^{(l)}(t)$(the unit hydrograph for the subsystem) is derived by converting the AR process to the MA (moving average) process,

$$h_0^{(l)} = 0,\ h_1^{(l)} = 1/\beta^{(l)},\ h_2^{(l)} = a_1/\beta^{(l)},$$
$$\ldots,\ h_n^{(l)} = \Sigma h_{n-j}^{(l)} a_j \quad (n>1) \quad (6)$$

where for brevity the superscript (l) is omitted and β is a factor to be described subsequently.

2.3 Estimation of snowfall from snowmelt runoff

By operating the AR coefficients determined above on the original separated runoff time series $y^{(l)}(t)$ the time series of snowfall to the subsystem $x^{(l)}(t)$ is inversely estimated as the random input .

$$x_{t-1}^{(l)} = \beta^{(l)} \cdot [y_t^{(l)} - a_1^{(l)} y_{t-1}^{(l)} - a_2^{(l)} y_{t-2}^{(l)} - \cdots\cdots - a_p^{(l)} y_{t-p}^{(l)}] \quad (7)$$

where the conversion factor $\beta^{(l)}$ is given by

$$\beta^{(l)} = \frac{24*3.6}{A(1-a_1^{(l)}-a_2^{(l)}-\cdots-a_p^{(l)})} \quad (8)$$

in which the catchment area A is expressed in *unit of* km^2

, both y and ε in *unit of m³/s* and x in *unit of mm/day*.

Summing up all estimated components of subsystems, the time series of effective snowfall (Xi) is estimated by equ.(9).

$$X_t = \sum_{l=1}^{2} x_t^{(l)}, \qquad l=1,2 \tag{9}$$

3. APPLICATION TO A BASIN FOR SNOWMELTING PERIOD

The above mentioned method has been applied to the daily snowmelt runoff from the Yuda River basin with the catchment area $A = 583$ km^2 in Japan.

3.1 Examination of white noise of snowfall data

Since the energy of power spectral of daily snowfalls is assumed to be constant, that is, the auto-correlation function is Dirac's delta function as is clear from Fig. 1, the time series of snowfalls may be assumed to be of white noise.

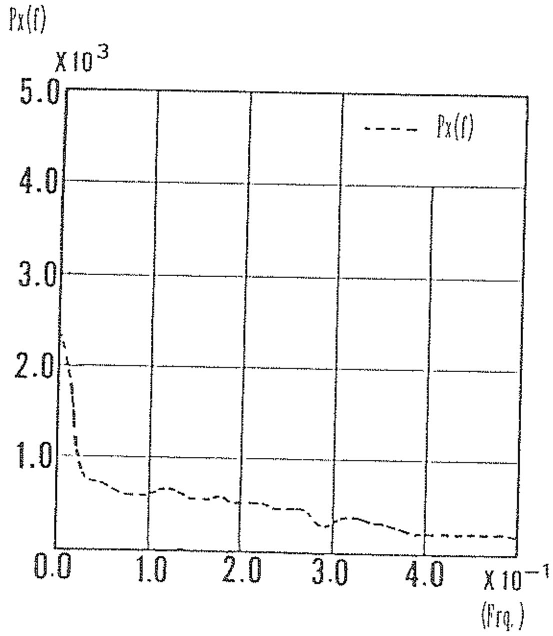

Figure 1 Power spectrum of snowfall (input)

3.2 Determination of cutoff frequency

An effective measure of estimating the cutoff frequency is provided by inspecting the coherence of the input (snowfall) and the output (snowmelt runoff)(Fig.2) which indicates that fc = 0.45 day^{-1} for the Yuda River

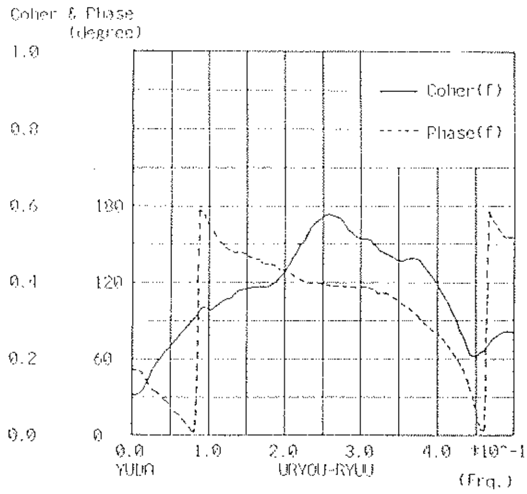

Figure 2 Coherence and Phase

Order of *AR coefficient*	*AR coefficients* of longer period	*AR coefficients* of shorter period
1	1.4177	0.6412
2	-0.7121	-0.0785
3	0.2287	0.1200
4	- 0.0992	----
5	0.0951	-----

Table 1 AR coefficients

3.3 Separation of daily snowmelt runoff.

By operating the high cutoff numerical filter and yet a damping factor δ is 2.0 , the snowmelt runoff data have been separated into the time series of two components, that is, the longer period component and the shorter period component (Fig.3).

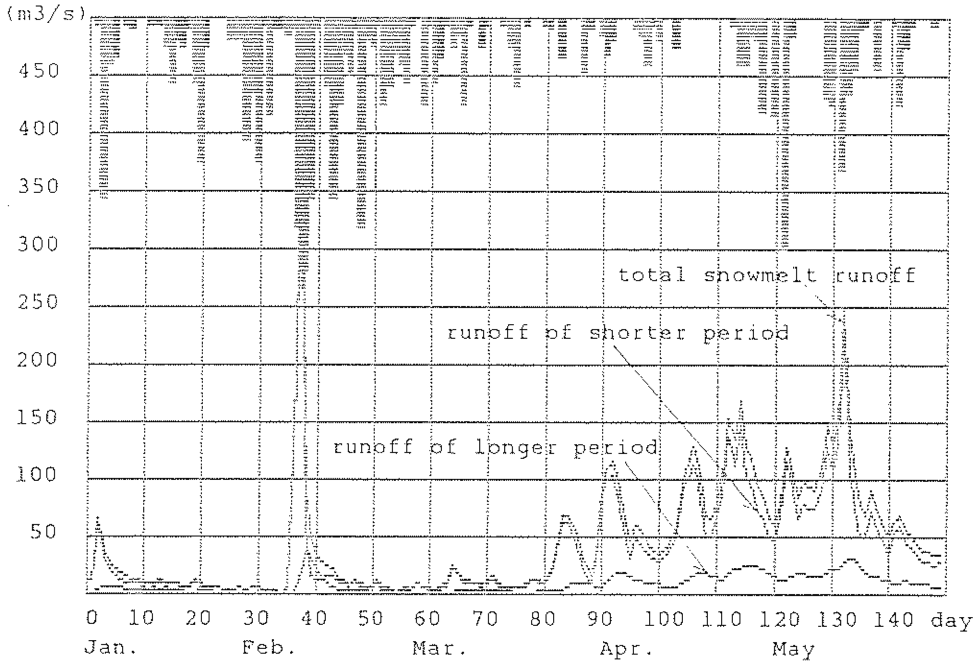

Figure 3 Separation of snowmelt runoff into components

3.4 Hydrologic characteristics (unit hydrograph)

By fitting the AR model, equ.(5), to the filtered time series, the longer period and the shorter period, the AR coefficients for each subsystem are determined with the result as shown in Table 1. The number of terms in optimal AR model fitting has been determined by the least squares criterion of the prediction error, for reference, the results of the other criteria of final prediction error and Akaike information criterion (Akaike,1976).

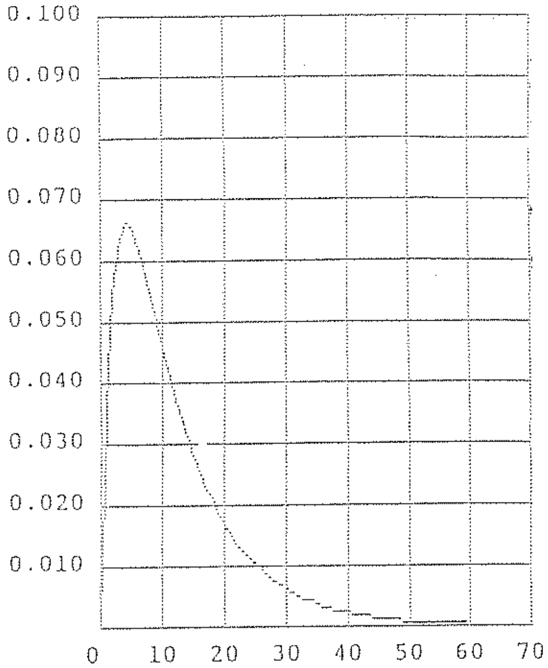

Figure 4(a) Response function of longer period

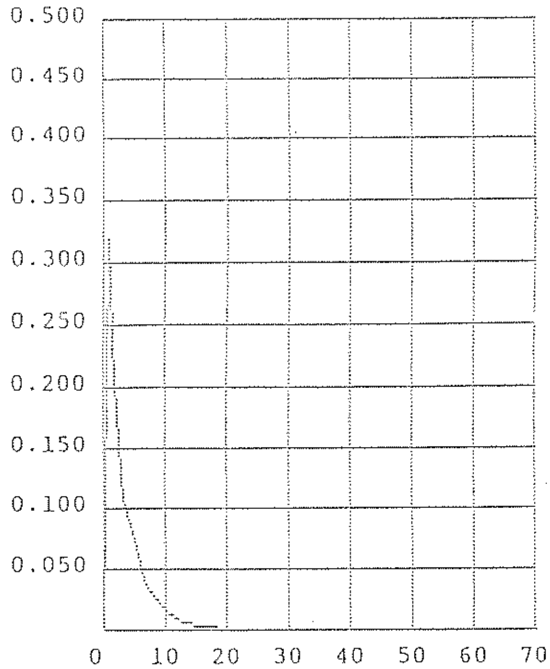

Figure 4(b) Response function of shorter period

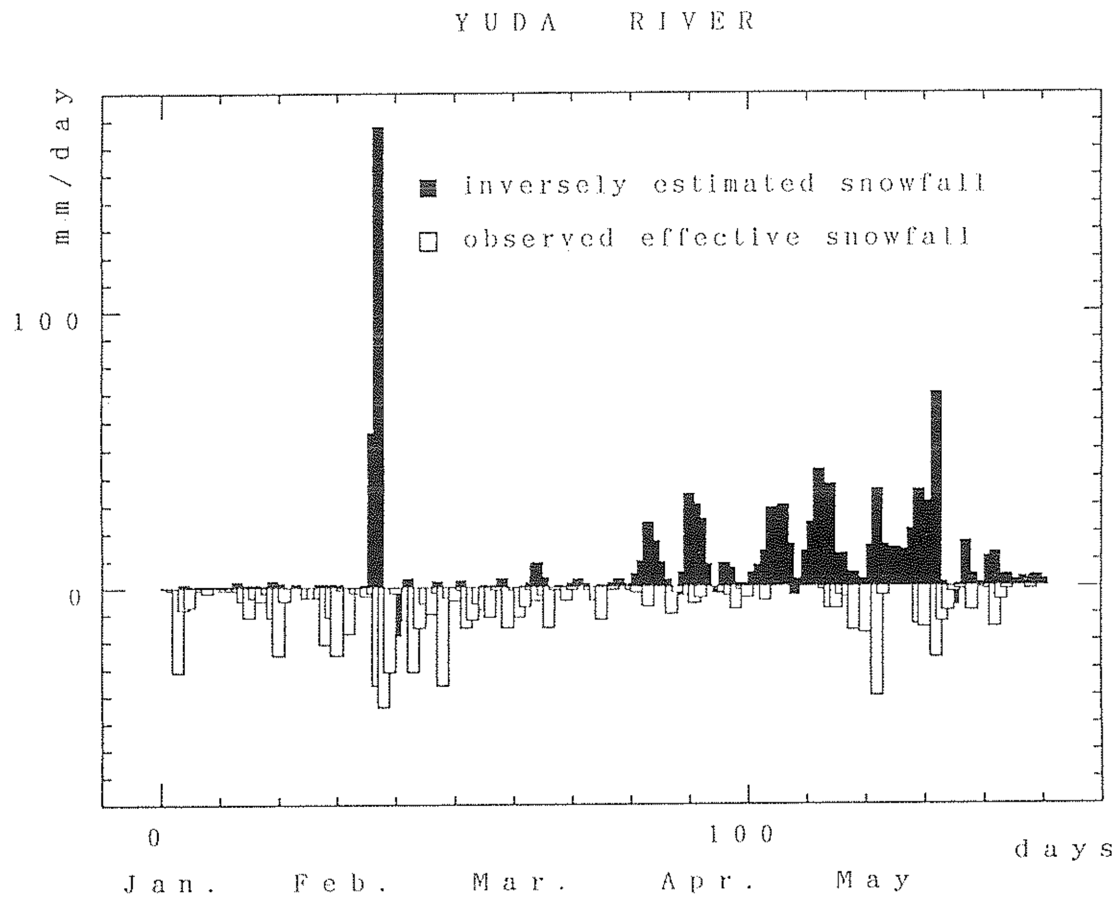

Figure 5 Comparison the inversely estimated snowfall with the observed effective snowfall

In this analysis, the snowmelt runoff component of shorter period is fitted by the third-order autoregressive model, while that of longer period is fitted by the fifth -order autoregressive model.

The unit impulse response function for the subsystem is obtained through equ.(6) and shown in Fig.4(a) and (b).

From Figs.4, it is understood that the snowmelt runoff of shorter period has a peak discharge at a lag of one day and ends within about twenty days. On the other hand, the snowmelt runoff of longer period has a peak discharge at about two days lag and lasts as long as about sixty five days. In the case of snowfall, during the snowmelting period, even if snow thaws, snowmelt water does not flow out to stream at once because snowmelt water has to first flow through the layer of drifted snow and soil (Hasebe and Kumekawa,1994a and 1994b).

3.5 Inverse estimation of effective snowfall

After the AR coefficients or the response functions of shorter period and longer period components were determined, the virtual snowfall component, which contributes to the actual snowmelt runoff, to subsystems as well as the composite effective snowfall have been inversely estimated by equ.(9).

In Fig. 5 the inversely estimated composite snowfall (Xi) is compared with observed effective snowfall which is obtained by multiplying the observed snowfall by the runoff ratio *f* (observed total snowfall/snowmelt total runoff) which is estimated from cumulative rvedsnowfall and cumulative snowmelt runoff for snowmelting period. The value *f* is 1.230.

Judging from figure, the snowmelt period is from the last eleven days of March to the end of May in the Yuda River.

4. CONCLUSIONS

The key to this hydrologic inverse problem lies in the fact that the input (daily snowfall) is of white noise and the physical process in the hydrologic subsystems are understood although they were only conceptual.

The conclusions of this analysis are as follows.

(1)The cutoff frequency is estimated from the coherence gap between snowfall and snowmelt runoff and the value *fc* is 0.45.

(2) The snowmelt runoff component of shorter period is fitted by the third-order autoregressive model, while that of longer period is fitted by the autoregressive model with the order of 5.

(3)The snowmelt runoff component of a shorter period has a peak discharge at a lag of one day and ends within about twenty days, while that of a longer period has a peak discharge at about two days lag and lasts as long as about sixty five days from the unit impulse response function of subsystems.

(4) Comparing the estimated virtual snowfall with the observed snowfall, it is understood that snowmelt water does not run directly off to the river as discharge; part of the snowmelt water infiltrates into the snow pack and soil, and the snowmelting period when snowmelt water flows out to the river is from the last eleven days of March to the end of May in the Yuda River from Fig.5.

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